HARVARD GRADUATE SCHOOL OF EDUCATION

Improving learning and engagement in algebra through *comparison* and *explanation* of *multiple strategies*

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Getting started/introductions

- Find one person in the room that you don't already know
- Introduce yourself
- Then share answers to the following questions:
  - What does it mean to teach math with *multiple strategies*? (It is OK if you don't know much about what this phrase means – we will be talking about this today.)
  - True/False: All math problems can be solved in more than one way. (Explain your answer)
  - True/False: Students should be able to solve math problems in more than one way. (Explain your answer)

Today's Agenda

- Think together about a math teaching scenario that helps motivate today's focus on *comparison* and *explanation* of *multiple strategies*
- Presentation from Jon about best practices for using *comparison* and *explanation* of *multiple strategies*
- Lesson planning and demonstration teaching to practice implementing *comparison* and *explanation* of *multiple strategies*

My goal for the day is ...

- You will develop familiarity with teaching techniques related to *comparison* and *explanation* of *multiple strategies*, including how, why, and when these techniques can be used to improve students' learning and engagement in mathematics, especially in algebra

To get started, let's think together about a math teaching scenario...

Alex and Morgan are students in an Algebra I class. They were asked to solve the following linear equation:

$$3(x + 2) + 4(x + 2) = 21$$

Here is Alex's method:

$$3(x + 2) + 4(x + 2) = 21$$
$$3x + 6 + 4x + 8 = 21$$
$$7x + 14 = 21$$
$$7x = 7$$
$$x = 1$$

Here is Morgan's method:

$$3(x + 2) + 4(x + 2) = 21$$
$$7(x + 2) = 21$$
$$x + 2 = 3$$
$$x = 1$$

Some questions for us:

1. Morgan says that her solution method is better than Alex's method. Why might Morgan believe that her way is better?
2. Alex disagrees and says that his solution method is better than Morgan's method. Why might Alex believe that his way is better?
3. Who do you think is correct in this argument between Alex and Morgan, about which way is better for solving this equation? Why?

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1. Morgan says that her solution method is better than Alex's method. Why might Morgan believe that her way is better?
2. Alex disagrees and says that his solution method is better than Morgan's method. Why might Alex believe that his way is better?
3. Who do you think is correct in this argument between Alex and Morgan, about which way is better for solving this equation? Why?

Is Alex's way better?

- It is better *for Alex*
- It always works, for lots of different kinds of equations
- It can be done quickly without a lot of thought
- He got the right answer, which is more important than whether his method was the best one

Is Morgan's way better?

- It is shorter, more efficient, faster
- It is clever, elegant, better matched to the specific features of this equation
- Morgan's use of this strategy indicates knowledge of multiple equation-solving strategies as well as understanding of algebra principles such as variable

Alex may think that:

- Math is all about memorizing formulas
- How I solve problems isn't important – all that matters is to *get the right answer*
- There is almost always only one right way to solve a math problem, and this one way is how the teacher does it
- Thinking and creativity don't play a big part of learning math
- As long as I can solve the problem, it doesn't matter if I understand what I am doing

Morgan may think that:

- Math is about *how* I solve problems as well as the answers I get
- There are many ways to solve math problems
- It is important to know which ways work and don't work, and which ways are clever and not clever, when solving math problems
- Thinking and creativity play essential roles in learning math
- Understanding math ideas goes hand-in-hand with knowing how to solve problems

My perspective …

- We want more students to be like Morgan
  - Too many of our students are like Alex
- ALL students are capable of being like Morgan
  - Not only stronger but also weaker students
- Students like Morgan will enjoy math more and also experience greater success
- *How we teach* math determines whether our students are more like Alex or more like Morgan
  - It is much harder to teach so that we produce students like Morgan

Teaching with multiple strategies

- Teaching with multiple strategies has the potential to:
  - Make math class more interesting and assessable
  - Deepen students' understanding of the math
- Core to teaching with multiple strategies requires effective use of
  - Comparison
  - Explanation

Educator's Practice Guides

- US Department of Education, Institute of Education Sciences
- Summarize best available research evidence on current challenges in education
- "Teaching Strategies for Improving Algebra Knowledge in Middle and High School Students"
- Three recommendations

Recommendation 1

- Use solved problems to engage students in analyzing algebraic reasoning and strategies
  - Discuss solved problems to make connections among strategies and reasoning
  - Select solved problems that reflect the lesson's instructional aim, including illustrating common errors
  - Use whole-class discussion, small-group work, and independent practice to introduce, elaborate, and practice working with solved problems

Recommendation 2

- Teach students to utilize the structure of algebraic representations
  - Promote use of mathematical language
  - Encourage students to use reflective questioning to notice structure
  - Teach students that different algebraic representations can convey different information about a problem

Recommendation 3

- Teach students to intentionally choose from alternative algebraic strategies when solving problems
  - Teach students to recognize and generate strategies
  - Encourage students to articulate the reasoning behind their choice of strategy and the mathematical validity of their strategy
  - Have students evaluate and compare different strategies for solving problems

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Comparison

- Is a *fundamental learning mechanism*
  - We learn by comparing and contrasting
- Strong research evidence support for benefits of comparison from cognitive science
- Helps focus our attention on critical features of what we are trying to learn
- For example...

Buying a TV at Best Buy

- How do I decide which TV to buy, and how does comparison help?
- Let's say I start with price and pick two TVs at the same price that both look OK

Sony 55", $999.99

LG 55", $999.99

Look at TV #1 (Sony 55”)

Sony 55” Information

Sony 55”, $999.99

Look at TV #2 (LG 55")

LG 55" Information

LG 55", $999.99

Limits of sequential viewing

- Hard to remember information about the Sony when looking at the LG and vice versa
- Hard to know which features the TVs are similar on and which they are different on
- As a result, I am just as likely to be influenced by *superficial* features (shape and color) than by ***substantive*** features

What if I compare?

Comparing these two TVs

Sony 55”, $999.99

LG 55”, $999.99

Comparison

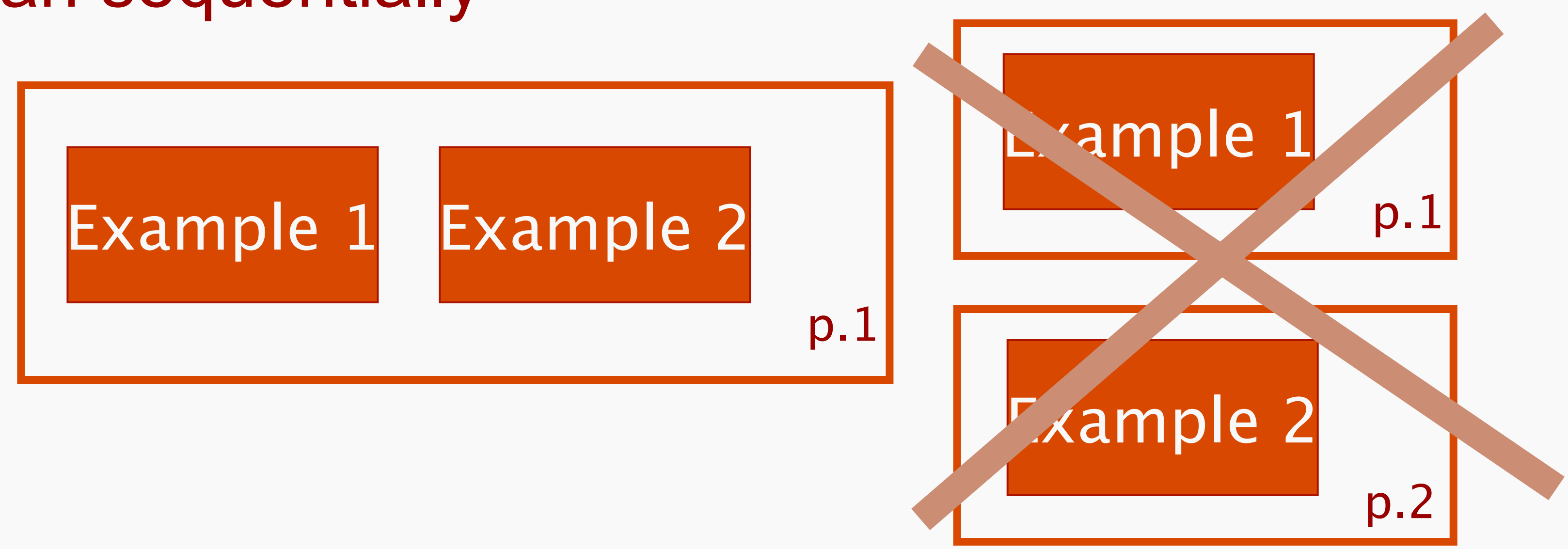
- Makes it easier for me to see the features where these TVs are the same and different
- In the case of algebra learning...

Comparison helps students...

- See similarities and differences between problems, representations, and strategies
- See how and why a strategy works for a particular problem
- See how and why a representation is particularly useful for answering certain kinds of questions
- See why some strategies or representations are better than others for certain kinds of problems

Carefully designed visuals

- Instructional materials and presentation should provide opportunities for students to see multiple problems and strategies at the same time, rather than sequentially



Can take several formats

- Display the same problem solved two different ways, side by side
- Display two different problems solved in the same way, side by side
- Display two different problems solved in two different ways, side by side
- All of these ways of presenting problems are better for student learning than sequential presentation

Typically...

Problem #1

- Students see one worked out problem

Problem #2

- Followed by another one on a different page
- Hard to compare these two strategies when presented in this way
  - Think back to the TV example

What if we compared?

Problem #1	Problem #2

- But when presented together, we can compare much more easily

Supporting comparison

- Teachers need to provide visual and gestural cues to aid comparison
  - Evidence from studies of international math teaching and learning that teachers in high performing counties do more of this than US teachers
- Horizontal back-and-forth indicates comparison is happening
- Teachers' gestures can provide support for comparison

Incorporating explanations

- Provide opportunities for students to compare and evaluate different strategies that they are learning in a discussion via explaining to partners and peers
- Two types of explanations that are helpful:
  - Similarities and differences in strategies
  - Evaluation of strategies

Similarities and differences

- How are these strategies similar?
- How are they different?
- How are these strategies related to other strategies that we have used before?

Evaluation of strategies

- Which strategy is better for this problem and why?
- Why is this strategy the most effective (or most efficient, or most elegant, or the best) to use for this problem?
- On what kinds of problems is this strategy most effective (or most elegant, or the best)?

Teacher questioning is key

Example 1.1. Questions to facilitate discussion of solved problems

- What were the steps involved in solving the problem? Why do they work in this order? Would they work in a different order?
- Could the problem have been solved with fewer steps?
- Can anyone think of a different way to solve this problem?
- Will this strategy always work? Why?
- What are other problems for which this strategy will work?
- How can you change the given problem so that this strategy does not work?
- How can you modify the solution to make it clearer to others?
- What other mathematical ideas connect to this solution?

Teaching with *Multiple Strategies* Checklist

Comparison:

- Teaching Materials:
  - ❑ Two solved problems, side by side
  - ❑ Problem steps aligned
  - ❑ Problems that are mostly similar but different in ways that align with the instructional objective
  - ❑ Materials illustrate multiple approaches, common errors, or key ideas
  - ❑ Not too busy!

Teaching with *Multiple Strategies* Checklist

Comparison:

- Instruction:
  - ❑ *Understand*, then *Compare*, then *Make Connections*
  - ❑ Visual and gestural supports for comparisons made
  - ❑ Clear lesson closure

Teaching with *Multiple Strategies* Checklist

Explanation:

- ❑ Varied grouping structures to give all students opportunity to participate in discussions and to explain
- ❑ Open-ended questions asked, especially questions that ask students to articulate the reasoning behind their choice of strategies and the mathematical validity of their strategies
- ❑ Similarities, differences, and evaluation comparisons made

What might teaching with *comparison* and *explanation* of *multiple strategies* look like in an actual lesson?

Demo mini-lesson

- Jon will try to teach a mini-lesson as if you were his Algebra I students
- Half of you will be the students in Jon's class; the other half are observers
- Note that the goal here is for Jon to provide a typical lesson using *comparison* and *explanation* of *multiple strategies*, not an exemplary lesson
- This lesson will show one way that Jon tries to implement the principles in the previous slides

Roles

- "Students": Play along and act as if you were one of your own students, not by misbehaving but by asking the kinds of questions that your students might ask about the lesson material and/or by making the kinds of errors that your students tend to make. Be nice! You may need paper and something to write with.
- Observers: Be aware of Jon's teaching and whether he is accomplishing his goals for comparison and explanation of multiple strategies

About the curriculum I'll use

- Supplemental Algebra I curriculum
- Designed by me and my collaborators at Harvard and Vanderbilt
- Intended to support teachers' effective use of *comparison* and *explanation* of *multiple strategies*

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Let's begin!

Do now!

- Solve:

$$\begin{cases} 4x + 5y = -1 \\ 3x + 2y = 1 \end{cases}$$

Debrief!

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Your turn…

- Get into groups (suggested size ≤ 5)
- Pick a topic/lesson from anywhere in the high school math curriculum
- Design a short demo lesson that uses *comparison* and *explanation* of *multiple strategies*
  - Make materials and a lesson plan
  - Consult the checklist
- Then each group will teach your lesson, and we will debrief afterwards

Useful Resources

- Practice Guide for Algebra
- http://ies.ed.gov/ncee/wwc/PracticeGuide/20

- Algebra I materials with Alex and Morgan
- http://scholar.harvard.edu/contrastingcases/book/materials-0

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Thanks! Stay in touch!

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